Estimation of Hydrodynamic Parameters of Soils in the Unsaturated Zone of Allada Plateau by Scaling Infiltration Equation and Beerkan Methods

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Summary

The study of the effect of the variability of hydrodynamic parameters of soils in the unsaturated zone on flux requires the prior accurate estimate of hydrodynamic parameters. At local scale, traditional methods of physical and hydrodynamic characterization of soils are very accurate and efficient to describe the transfer of water in the soil. Just only the hydrodynamic properties of the soil are sufficiently well known. At large scale against, these methods have their limitations. They prove in fact, often time consuming, and larger areas become difficult to be characterized at hydrodynamic point of view.

For parameterization of the main soils of the Allada plateau consists of Haplic Acrisols, Umbric Fluvisols and Ferric Acrisols, we adopted a simplified method, easy to implement and much less time and equipment consuming. This is an experimental method called Beerkan method which relies on the knowledge of the particle-size distribution of the soil, its dry density, simplified infiltration tests using cylinders and scaling models of infiltration equations used as references. This study describes the experimental protocol and method of analysis which is applied to the estimation of parameters from the retention curve models, hydraulic conductivity and infiltration equation describing the hydrodynamic properties of soils of the unsaturated zone. The parameters of two orders are the hydrodynamic shape parameters and scale parameters.
The shape parameters related to the soil texture are derived from particle-size data analysis. The scale or normalization parameters depend on soil structure. They are obtained by inverse modeling and optimization from the infiltration tests. The concept of sorptivity and non-dimensional equations of the infiltration are the theoretical bases of this study. The results from the formalism for infiltration mono- and three-dimensional of Braud study (2005) are used. The relevance of the method has been shown from in situ and laboratory measured data on the main soils of the study area aforementioned. To adjust the observations to models, we adopted a simple optimization method applied to two boundary models, the Green & Ampt (1911) and Talsma & Parlangue (1972) models. At the optimization step, two (02) fitting programs written in R language have allowed one to optimize shape parameters and the other the scale parameters. Fitting results of some Haplic Acrisols and Ferric Acrisols are satisfactory with the GA model contrary to the TP model that has not reproduced well the hydrodynamic behavior of soils of the study area. The fitting error RMSE of all the soil is less than 15%. But model and data curves are not well fitted excepting ferric Acrisols of SOCIII_50 site.

**Keywords:** unsaturated zone of Allada Plateau, Beerkan methods and scaling, shape and scale parameters, texture and structure, inverse modeling and optimization, mono-infiltration and three-dimensional and non-dimensional equation

**Introduction**

The unsaturated zone, sometimes called the vadose zone, is the zone between the ground surface and the water table. The term unsaturated zone is somewhat of a misnomer as the capillary fringe above the groundwater or the rain-saturated top soil are portions that are saturated. For this reason some authors (e.g., Bouwer, 1978) prefer the term vadose zone.

This zone study has several applications, for example in domains of agriculture, civil engineering and environment, management of water and soils resources as infiltration is often followed by pollutants transfer (de Condappa et Soria, 2002).

The unsaturated zone plays a crucial role in the transfer of pollutants. This zone is the hydrological connection between the surface water component of the hydrologic cycle and the groundwater component (Haverkamp et al., 1999). Many constituents present in the surface waters eventually find their way into the groundwater through the unsaturated zone. Accident spills of chemicals, application of fertilizers and pesticides on the land surface, leaks from gasoline storage tanks, septic tank drainage, leaching from landfills are examples of anthropogenic activities that contribute to the leaching of contaminants through the unsaturated zone into the groundwater.

It is therefore important to describe accurately water movement within the unsaturated zone in order to quantify properly the various components of the hydrological cycle.

Water movement within the unsaturated zone is often described by the formalism proposed by Richards (1931).

The knowledge of hydrodynamics parameters and therefore, the hydrodynamic behavior of soils in unsaturated zone pass by the resolution of flow equation of Richards. To obtain these parameters required to solve flow equation, one must specify the initial and boundary conditions, as well as functions relating the soil water pressure and the hydraulic conductivity to the water content (retention and hydraulic conductivity curves).

These curves can be described by shape and normalization parameters, linked to soil texture and soil structure, respectively, the identification of which has led to an abundant literature (see, for example, Haverkamp et al. (1998a) for a review). Through the Beerkan method, this study show how these parameters can be identified at the local scale from simple infiltration tests, measurements of dry
bulk density and particle-size analysis. The method is applied to the following main soils from unsaturated zone of Allada plateau, namely haplic Acrisols, umbric Fluvisols and ferric Acrisols.

Furthermore, according to Braud (2005), the methodology is simple enough to be applied at many locations so as to estimate the spatial variability of these properties. Braud et al. (2005) provide an example of such application for a 100 km x 100 km area in central Spain, sampled every 1 km in 1994, using a pioneering version of the method. The technique has also been used to map the hydraulic properties in the topsoil of a 580-km² catchment in Benin (de Condappa, 2005), based on infiltration tests every 0.1°. This author underlined that papers presenting the theoretical background of the method or the experimental protocol in detail have still been lacking.

This study, as previous authors, spilled the methodology by applying it to the specific case of Allada plateau soils. The paper is organized as follows. In the first part, it gives the theory to derive the shape parameters from textural data of the three main soils of Allada plateau. The study is primarily to show the feasibility and soundness of the approach, and as Braud et al. (2005), we believe that the use of bounding cases is sufficient. The aim of the study is to give the first assessment of the accuracy of the Beerkan method and the steps of the data analysis are presented in the second part, and, in the third part, we give a first assessment of the accuracy of the Beerkan method using infiltration data measured in the three main soils of Allada plateau mentioned above for one- and three-dimensional infiltration. It illustrates the theory with a simple optimization technique and applies this latter to two bounding cases, providing the range for the estimated parameters. The methodology is applied to the following main soils from Allada plateau soils. The paper is organized as follows. In the first part, it gives the theory to derive the experimental protocol of the Beerkan method and the steps of the data analysis are presented in the second part, and, in the third part, we give a first assessment of the accuracy of the Beerkan method using infiltration data measured in the three main soils of Allada plateau mentioned above for one- and three-dimensional infiltration. It illustrates the theory with a simple optimization technique and applies this latter to two bounding cases, providing the range for the estimated parameters. The aim of the study is to show the feasibility and soundness of the approach, and as Braud et al. (2005), we believe that the use of bounding cases is sufficient.

### Theoretical Backgrounds

#### Description of Water Movement within the Unsaturated Soil

Soil water movement in unsaturated homogeneous soil (such as infiltration, evaporation and drainage) is supposed to be isothermal and one-dimensional. It can be expressed through the partial differential equation (Richards, 1931) as follows.

\[
\dot{c}(\theta) \frac{\partial h}{\partial t} = \frac{\partial}{\partial z} \left[ K(\theta) \left( \frac{\partial h}{\partial z} - 1 \right) \right],
\]

where \(\theta\) is volumetric water content, \(h\) is soil water pressure (m) relative to atmospheric pressure (h\(\leq\)0), \(K(\theta)\) is hydraulic conductivity (m/s) as a function of \(\theta\), \(z\) is depth (m) taken as positive downwards, and \(t\) is time (s), \(C(\theta)\) is the hydraulic capacity (m\(^{-1}\)) as given by \(C(\theta) = \frac{d\theta}{dh}\).

The solution of Equation (1) subject to given initial and boundary conditions describes the evolution of the water content profiles \(\theta(z, t)\) as function of depth and time.

The complete solution of the Richards equation requires the knowledge of at least seven parameters in the large sense (one boundary condition, one initial condition and generally five parameters describing the hydraulic properties, see below). According to the pioneering authors of Beerkan method, a first step in our analysis should be to reduce this number of parameters to simplify the identification problem.

#### Representation of Soil Hydraulic Properties

Soil hydraulic properties are represented as mathematical functions.

Many different functions have been proposed in the literature based on various combinations of the dependent variables \(\theta, R, \) and \(K\), and a certain number of fitting parameters (e.g., Gardner, 1958; Brooks and Corey, 1964; Brutsaert, 1966; van Genuchten, 1980; Haverkamp and Vauclin, 1981).

This study presents only the most commonly used models. For the water retention curve, the Brooks & Corey (1964) and van Genuchten (1980) models are often considered:

\[
\frac{\theta}{\theta_s} = \left( \frac{h}{h_{BC}} \right)^{\lambda}
\]

and

\[
\frac{h}{h_{BC}} = \left[ 1 + (\lambda \theta_s)^{\lambda} \right]^{\frac{1}{\lambda}}
\]
\[
\frac{\varepsilon}{\varepsilon_0} = \left\{ 1 + \left( \frac{\theta}{\theta_0} \right)^{\lambda} \right\}^{\frac{1}{n}}
\]

(3)

With \[ \eta = \frac{k}{1-m} \]  

(4)

where \( \theta_0 \) is the saturated water content, \( k_{BC} \) and \( k_{VG} \) are normalization parameters for the water pressure, \( \lambda \), m and n are shape parameters and k is an integer chosen to be 1 (Mualem, 1976) or 2 (Burdine, 1953). As the parameter values of m and n depend on the choice of k, Haverkamp et al. (2004) developed a routing method that allows one to pass from k=1 to k=2 and vice versa (Braud et al., 2005).

For the hydraulic conductivity curve, the study used the Brooks & Corey (1964) model:

\[
K(\theta) = K_s \left( \frac{\theta}{\theta_s} \right)^\eta
\]

(5)

where \( K_s \) (m/s) is the saturated hydraulic conductivity, \( \eta \) is the shape parameter and the other parameters are defined above.

The shape parameters are related dominantly to soil texture (Haverkamp et al., 2002), while the three normalization parameters: saturated water content \( \theta_s \), saturated hydraulic conductivity \( K_s \), and the normalization parameter for water pressure (\( h_{BC} \), \( h_{VG} \)) or \( h_f \) depend mainly on soil structure (Haverkamp et al., 1998a).

Consequently, the full description of the Richards equation requires the knowledge of six parameters, which comprises the four soil parameters identified above and the initial and boundary conditions.

**The Concept of Sorptivity**

Sorptivity \( S_1(\theta_0, \theta_1) \) is a physical characteristic specific to the soil which translates, under the initial condition \( \theta_0 \) and the boundary condition \( \theta_1 \), its capacity to absorb water when gravity is neglected (de Condappa, 2005).

For practical convenience, we use here the simple but accurate (Elrick & Robin, 1981) estimation of sorptivity given by Parlange (1975):

\[
S_1^2(\theta_0, \theta_1) = \int_{\theta_0}^{\theta_1} \left( 1 + 2 \theta - 2 \theta_0 \right) D(\theta) d\theta
\]

(6)

where \( D(\theta) \) is the diffusivity (m²/s) defined by \( D(\theta) = \frac{K(\theta)}{\theta_0} \frac{dh}{d\theta} \)

The integral given by Equation (7) from 0 to \( \theta_1 \) of \( S_1(\theta_0, \theta_1) \) represents the whole range of possible soil moisture content. It does not depend on initial and boundary conditions and can be considered as an integral soil characteristic.

The concept of *sorptivity* represents an important integral variable which links water retention and hydraulic conductivity characteristics.

\( S_1(\theta_0, \theta_1) \) can be calculated analytically for the various retention models presented in the previous section (combination of either the Brooks & Corey or the van Genuchten model for the retention curve, with the Brooks & Corey model for the hydraulic conductivity model) as follows.

\[
S_1^2(\theta_0, \theta_1) = -c_p K_s \theta_s h_{\text{norm}}
\]

(7)

where \( h_{\text{norm}} \) refers to \( h_{BC} \) of the Brooks & Corey model, Equation (2), \( h_{VG} \) from the van Genuchten model, Equation (3), and \( h_f \) from the GA model; and \( c_p \) is a texture only dependent factor, the expression for which is given by Haverkamp et al. (1998a) for the Brooks & Corey and van Genuchten models.

For the GA model, \( c_p = 2 \) and \( h_{\text{norm}} = h_f \)

When a positive head \( h_{\text{surf}} \) is applied at the surface, the sorptivity expression must be modified to account for it and this leads to (Haverkamp et al., 1998a):

\[
S_1^2(\theta_0, h_{\text{surf}}) = S_1^2(\theta_0, \theta_2) + 2K_s h_{\text{surf}} (\theta_2 - \theta_0)
\]

(8)
Infiltration Equation

Various authors (e.g., Philip, 1969; Parlange, 1985; Haverkamp et al., 1990) have developed analytical solutions for the Richard’s equation considering one-dimensional constant-head infiltration $h_{surf}$ in a semi-infinite soil column with constant initial volumetric water content $\theta_0$.

Second integration of the soil moisture profile $\theta(z,t)$ over depth $z$ provides the soil moisture storage and can give access to the one-dimensional cumulative infiltration, $I_1(m)$ (when the surface flux is directed downwards), as a function of time:

$$I_1(t) = h_0 t + \int_0^t [\theta(z, t) - \theta(z, 0)] \, dz$$

Integrating over depth results in one degree of freedom being removed, and the infiltration can be described with only four parameters, two of them being the initial and boundary conditions (Braud et al., 2005).

If the latter are known, then infiltration can be used to identify the remaining two unknown normalization parameters, as is shown below.

General form of the Infiltration Equation for a Concentration Type Boundary Condition

The general form of infiltration equation, equation (10) considered in this study is that of Haverkamp et al. (1990). They derived it from the particular case of a uniform initial condition $\theta(z, 0) = \theta_i$ and a constant head infiltration $h_1$.

The constant water pressure value $h_1$ can be negative or positive equal to the ponded water depth ($h_{surf}$) imposed at the soil surface ($h_2 = h_{surf} \geq 0$).

$$I_1(t) = h_0 t + \frac{h_{surf} (\theta_0 - \theta_i)}{\theta_i - \theta_0} + \frac{S_z^2 (\theta_0 - \theta_i)}{2 (\theta_i - \theta_0) \beta} \ln \left\{ 1 + \beta \frac{(h_2 - h_1)}{\theta_i - \theta_0} \right\}$$

(10)

where $\beta$ is an integral parameter, $\beta \in [0,1]$, depending on soil characteristics, initial and boundary conditions, and with the other parameters defined as above. The parameter $\beta$ is defined as a function of volumetric water content, $\theta_i \leq \theta \leq \theta_0$, as a result $\beta$ is influenced only by the surface boundary condition when $\theta_1 \leq \theta_i$ or $h_1 \leq 0$. Ross et al. (1996) showed that $\beta$ is slightly affected by changes in the surface boundary condition when $h_1$, especially when $\theta_1$ stays close to $\theta_i$ ($\theta_1 \geq 0.75 \theta_i$).

Following the definition given by equation (11) below, $\gamma$ corresponds to the relative sorptivity fraction calculated over the saturated domain.

$$\gamma = \frac{h_{surf} K_0 (\theta_0 - \theta_i)}{S_z^2}$$

(11)

In summary, for general field soils $\gamma$ accounts for the effect of the positive head boundary condition on the infiltration process, e.g. $\gamma = 0$ when $h_{surf} = 0$. With this definition, Equation (10) can be rewritten according to Braud et al. (2005) as follows:

$$I_1(t) = h_0 t + \frac{S_z^2 (\theta_0 - \theta_i)}{2 (\theta_i - \theta_0)} + \frac{(1-\gamma) S_z^2}{2 (\theta_i - \theta_0) \beta} \ln \left\{ 1 + \beta \frac{(h_2 - h_1)}{\theta_i - \theta_0} \right\}$$

(12)

Note that if $h_1 < 0$, $\gamma = 0$ and $S_z^2$ must be replaced by $S_z^2 (\theta_0, \theta_i)$.

Haverkamp et al. (1998b) showed that equation (12) could be simplified and expressed as a function of $\beta (1-\gamma) \in [0, 1]$. Two particular cases of equation (12), representing the bounds for $\beta (1-\gamma)$, can therefore be considered. They are as follows.

1. If $\beta \to 0$, the equation reduces to the extended GA equation (with $K_0 \neq 0$).

$$I_1(t) = K_0 t + \frac{(\theta_0 - \theta_i) h_{surf} - h_2 (\theta_0 - \theta_i) K_0}{K_0 - K_2} \ln \left\{ 1 + \frac{(I_1(t) - h_{surf} - h_2 (\theta_0 - \theta_i)) K_0}{(\theta_0 - \theta_i) h_{surf} - h_2 (\theta_0 - \theta_i) K_0} \right\}$$

(13)

Or

$$I_1(t) = K_0 t + \frac{S_z^2}{2 (\theta_i - \theta_0) K_0} \ln \left\{ 1 + \frac{2 (I_1 - h_2 (\theta_0 - \theta_i)) K_0}{S_z^2 (\theta_0 - \theta_i)} \right\}$$

(14)
The fact that sorptivity, as given by equations (7), is a soil characteristic ensures that \( \beta \), introduced in Equation (14), is the same whatever the retention model used (for a given initial soil water content \( \theta_0 \)).

2 If \( \beta \rightarrow 1 \), equations (13) and (14) reduce to the Talsma & Parlange (1972) equation (TP below), which can be rewritten as:

\[
I_s(t) = \frac{\xi^2}{2(\theta_s - \theta_0)} \ln \left( 1 + \frac{\theta_s - \theta_t}{\theta_s - \theta_0} \right)
\]

(15)

These two limiting cases provide the bounds for any infiltration case (where \( \beta(1 - \gamma) \in [0, 1] \)), with uniform initial soil moisture content and positive or zero head at the surface.

**Expression of Three-Dimensional Infiltration**

In situ infiltration, especially when cylinders are used, is generally three dimensional due to effective lateral dispersion of water in the soil during the experiment. Haverkamp et al. (1994) proposed a theoretical three-dimensional solution based on the formulation of the one-dimensional infiltration equations, which takes into account the lateral infiltration. For a positive surface head \( h_{surf} \), the equation reads

\[
I_s(t) = I_1(t) + \frac{\phi^a S}{r(\theta_s - \theta_0)} t = I_1(t) + \delta t,
\]

(16)

Where

\[
\delta = \frac{\phi^a S}{r(\theta_s - \theta_0)}
\]

(17)

and \( r \) (m) is the radius of the infiltration device. The expression for \( I_1(t) \) was given in the previous section and the parameter \( \phi \) is confined in the interval [0.6, 0.8]. Haverkamp et al. (1994) showed that a value for \( \phi = 0.7 \) can be used in practice. The subscripts 1 and 3 refer to one- and three-dimensional infiltration, respectively. Parameter \( \beta \) (m/s), includes the effect of the axisymmetric water movement and the parameter \( \phi \) translates the effect of gravity in the three-dimensional infiltration.

**Scaled Form of the Infiltration Equation**

According to Birkhoff (1960), the scaling analysis of infiltration equation relies on the fundamental postulate that the invariance of a physical law (e.g. the vadose zone transfer equation) under a series of scale transformations implies the invariance of all consequences of the law under the same transformations (Braud et al., 2005).

This principle provides, consequently, the basis for dynamic similarity in the behaviour of two systems governed by the same physical law (Sposito & Jury, 1985).

Haverkamp et al. (1998a, b) applied inspectional analysis to the one-dimensional infiltration equation. Their results are extended here to the three-dimensional infiltration equation.

To do so, the following dimensionless variables (denoted with a * superscript) are defined.

\[
\Delta I_2 = I_2(t) - (K_o + \beta) t = \alpha_2 \Delta I_2^*,
\]

(18)

\[
t = \alpha_t \Delta t^*
\]

(19)

\[
\alpha_t = \frac{\Delta t^*}{2(K_s - K_o)}
\]

(20)

\[
\alpha_\theta = \frac{\Delta \theta^*}{2(K_s - K_o)} = \frac{\alpha_2}{\alpha_t}
\]

(21)

For the generalized infiltration equation of Haverkamp et al. (1990), the scaled equations read

\[
\Delta I_2^* = \frac{\gamma}{\Delta \theta^*} + \frac{1 - \gamma}{\beta} \ln \left[ 1 + \frac{\beta}{\Delta \theta^*} \right],
\]

(22)

\[
t^* = \frac{\gamma}{\Delta \theta^*} + \frac{1 - \gamma}{\beta(1 - \gamma)} \ln \left[ 1 + \frac{\beta}{\Delta \theta^*} \right] - \frac{1 - \gamma}{(1 - \beta)} \ln \left[ 1 + \frac{1}{\Delta \theta^*} \right]
\]

(23)
In the previous sections, the following two limiting cases were considered.

1. If \( \beta \rightarrow 0 \), the scaled form of the three-dimensional extension of the Green & Ampt (1911) equation reads:

\[
T^* = \Delta I^* - \ln \left(1 + \Delta I^* \right)
\]

2. If \( \beta \rightarrow 1 \), the scaled form of the 3D equivalent of the Talsma & Parlane (1972) equation reads:

\[
T^* = \Delta I^* - 1 + \exp \left(-\Delta I^* \right)
\]

As above, these two limiting cases provide the bounds for any infiltration case, with uniform initial soil moisture content and positive or zero surface head. In practice, we use these two equations as they represent the lower and upper limits for any infiltration case at uniform initial soil moisture content.

While the first model considers only the effect of gravity on water movement, the latter considers only capillary effects. The Beerkan method (below) provides the most efficient and cost-effective way to estimate soil scaling factors and soil hydrodynamic properties at a large number of sampling points. The proposed methodology relies mainly on simplified infiltration tests as well as particle-size analysis. Data analysis relies on the scaling theory presented in this section, and parameters are estimated for the two limiting cases provided by the GA and TP solutions. The estimated parameters will provide the confidence interval for the true values.

The Beerkan Method

The method is applied to experimental data obtained from three main soils of the study area:

- the Haplic Acrisols denoted ACho
- the Umbric Fluvisols denoted Flu
- and the Ferric Acrisols denoted ACfo

The method detailed with data collection and analysis allows assessing its accuracy from the use of simple optimization algorithm.

The experimental method was initially pioneered during the EFEDA experiment in Spain (e.g. Braud et al., 2005).

Experimental Set-up

The Beerkan method is a simple three-dimensional infiltration test under positive head conditions, made in cylinders of diameter \( r \) (ranging from 5 to 15 cm). These tests provide cumulative infiltration as a function of time, \( I_0(t) \). Measurements of particle size, initial and final water content, and dry bulk density are also required. The procedure is carried out in consecutive operational steps as follows.

1. The cylinder is positioned at the soil surface and inserted to a depth of about 1 cm into the topsoil, to prevent lateral losses of water. The surface vegetation is removed, while the roots remain in situ.
2. A disturbed soil sample is collected (0-5 cm depth) close to the cylinder. The initial gravimetric moisture content \( w_0 \) is estimated on this sample. Particle-size analysis is carried out on another disturbed sample taken at the experimental site.
3. Infiltration test. A fixed volume of water (approximately 100 ml) is poured into the cylinder at time zero, and the elapsed time during the infiltration of the known volume of water is measured. When the first volume has completely infiltrated, a second known volume of water is added to the cylinder, and the time needed for this to infiltrate is measured (cumulative time). The procedure is repeated for a series of about 8-15 known volumes and the cumulative infiltration time recorded. Note that with the procedure used, the surface pressure is no longer constant, as it is assumed in the theory above. However, Haverkamp et al. (1998a) showed that small variations of \( h_{surf} \) did not influence the results significantly.
4. A disturbed soil sample is collected (0-5 cm depth) within the perimeter of the cylinder, to obtain the final gravimetric moisture content \( w_f \) at the end of the infiltration test.

5. An undisturbed sample is taken within the wet soil inside the cylinder, in a cylinder of known volume to get the soil dry bulk density \( \rho_d \).

6. After the infiltration test, the approximate penetration depth of the wetting front and the range of lateral wetting are determined by manual removal of the wetted soil below the cylinder.

### Data Analysis and Algorithms

Particle-size analysis and derivation of shape parameters of the retention and hydraulic conductivity curves

The particle-size distribution is determined on a disturbed soil for the fine fraction (<2 mm). The particle-size distribution was fitted to the following equation which is a function of particle diameter \( d \) (m):

\[
F(d) = \left(1 + \left(\frac{d}{d_s}\right)^N\right)^{-M}
\]

with \( M = 1 - \frac{2}{N} \)

where \( d_s \) (m) is the normalization parameter for the particle diameter, and \( M \) and \( N \) are the shape parameters of the curve.

The quantities \( d_s \) and \( N \) are optimized by least squares techniques, stability of the convergence being achieved by a change of variable \( x = 1/d \). The shape parameters \( m \) and \( n \) of the water retention curve can be derived from the knowledge of soil texture, and more specifically, from the knowledge of the \( A \) and \( B \) values. Following Haverkamp et al. (2004), we introduce soil specific shape indexes, \( p_a \) for the particle-size distribution and \( p_m \) for the water retention curve, given by:

\[
p_a = \frac{N}{1 + N}
\]

\[
p_m = \lambda = \frac{MN}{1 + MN}
\]

where \( MN \) and \( MN \) are the products of the shape parameters of the retention and cumulative particle-size curves, respectively.

They are related through

\[
p_a = (1 + k) p_m
\]

where

\[
k = \frac{2s - 1}{2s_c(s - s_c)}
\]

and \( s \) is the solution of the non-linear equation below given by Fuentes et al. (1998):

\[
(1 - \epsilon)^2 + \epsilon^{2s} = 1
\]

where \( \epsilon \) is the soil porosity.

The derivation of the shape parameter for the hydraulic conductivity curve \( \eta \) is estimated from the classical capillary model

\[
\eta = \frac{2}{\mu_m} + 2 + \mu
\]

where \( \mu \) is a tortuosity parameter equal to 0 (Childs & Collis-George, 1950) or 1 (Mualem, 1976).

Shape parameters of the water retention and hydraulic conductivity curves can therefore be derived from sole knowledge of soil texture.

### Optimization of Normalization Parameters

Two normalization parameters can be derived from the dimensionless infiltration equation. In the following analysis, the saturated hydraulic conductivity \( K_s \) and the scaling factor \( \alpha_x \) are considered as independent parameters to be optimized by the simple infiltration experiment. To achieve this,
additional information on initial and boundary conditions is needed, namely the knowledge of $h_{surf}$ and $\theta_0$, as well as that of the third normalization parameter $\theta_s$.

The different steps of the analysis are summarized below.

1. The surface condition $h_{surf}$ can be estimated from the infiltrated volume $V$ (cm$^3$) and the radius of the cylinder, $r$ (cm):

$$h_{surf} = \frac{V}{\pi r^2}$$  \hfill (33)

but a value of $h_{surf} = 0$ can be used without greatly affecting the results.

2. The initial water content is obtained from the knowledge of the initial gravimetric water content $w_0$ and the dry bulk density $\rho_d$ (g/cm$^3$):

$$\theta_0 = \frac{w_0 \rho_d}{\rho_s}$$  \hfill (34)

If the dry bulk density has not been measured, the final gravimetric water content $w_f$ can be used as an approximate of the saturated gravimetric water content $w_s$ of the soil.

Zammit (1999) showed that the ratio $\theta_s/\epsilon$ can be approximated by $\theta_s/\epsilon = 2^m - \beta$, where the soil porosity $\epsilon$ is given by $\epsilon = 1 - \rho_d/\rho_s$ where $\rho_s = 2.65$ g/cm$^3$ is the soil particle density. The dry bulk density can be calculated from

$$2^m - \beta (\rho_s - \rho_d) = w_s \rho_d \rho_s$$  \hfill (35)

3. An estimate of the saturated water content can be derived from knowledge of the dry bulk density and the final gravimetric water content, which is assumed to be an approximation of the saturated gravimetric water content $w_s$:

$$\theta_s = w_s \rho_d$$  \hfill (36)

4. The initial hydraulic conductivity $K_0$ can be expressed as a function of the known quantities $\theta_0$, $\theta_s$, $\eta$ and the unknown parameter $K_s$:

$$K_0 = K_s \left( \frac{\theta_s}{\theta_0} \right)^\eta = a K_s$$  \hfill (37)

5. The next step is to define the function to be optimized.

Equations (22) and (23) provide the general dimensionless equations for three-dimensional infiltration. We showed that the GA and TP models, Equations (24) and (25), provide the upper and lower bounds for this general solution. Due to their simplicity, these equations are used in the optimization and provide upper and lower bounds for the estimated parameters.

Hence, we derive the optimized functions by introducing Equation (37) into Equations (24) and (25).

These expressions are rewritten, with $t$ as the dependent variable, to give Equations (38) and (36), respectively:

$$t(K_s, \alpha_t) = \frac{\Delta t_1}{K_s(1 - \alpha_t)} - \frac{\alpha_t}{K_s(1 - \alpha_t)} \ln \left(1 + \frac{\Delta t_1}{\alpha_t} \right)$$  \hfill (38)

$$t(K_s, \alpha_t) = \frac{\Delta t_1}{K_s(1 - \alpha_t)} - \frac{\alpha_t}{K_s(1 - \alpha_t)} \left(1 - \exp \left(-\frac{\Delta t_1}{\alpha_t} \right) \right)$$  \hfill (39)

Where

$$\Delta t_1 = I_3 - \left(a K_s + \delta(K_s, \alpha_t)\right) t,$$  \hfill (40)

and

$$\delta(K_s, \alpha_t) = \frac{2 a K_s (1 - \alpha_t) \alpha_t}{(\theta_0 - \theta_s)}$$  \hfill (41)

6. Optimization is performed subsequently by minimization of $F(K_s, \alpha_t)$, namely the sum of squared differences between modelled and observed times, $t_i$:

$$F(K_s, \alpha_t) = \sum_{i=1}^{N} \left(t_i(K_s, \alpha_t) - t_i \right)^2,$$  \hfill (42)
where \( t_i(K, \alpha) \) is the calculated time from either Equations (37) or (38) at the observation point \( (l_i, t_i) \) and \( n_{obs} \) is the number of observations.

Equation (42) can be solved (Marquardt, 1963) provided a relevant set of initial values is given for the parameter set \( (K, \alpha) \). This is all the more important because under three-dimensional infiltration the parameter is also unknown and depends on \( K \) and \( \alpha \).

Initial values for \( (K, \alpha) \) can be derived from a one-dimensional analysis of the data, where \( t \) is set to zero. For the illustrative purposes of this paper, the optimization procedure presented here will be sufficient.

7. Once \( K \) and \( \alpha \) are determined, the last step provides the value of the normalization parameter for water pressure with

\[
h_{\text{NORM}} = \frac{\eta}{c_p} \left( \frac{\alpha}{\eta} - \frac{h_{\text{surf}}}{1 - \alpha} \right)
\]

obtained from the expression of the sorptivity and the definition of \( \alpha \). Equations (7), (8) and (19). Note that the value of \( h_{\text{surf}} \) affects only the estimated \( h_{\text{NORM}} \), whereas the optimized values of the \( K \) and \( \alpha \) parameters are not affected by its choice.

Results & Discussions

This section presents the assessment of the accuracy of the optimization for the shape parameters and the normalization parameters using one-dimensional infiltration data and derived three-dimensional infiltration values of soils of the study area (Haplic Acrisol, Umbria Fluvisols and Ferric Acrisol).

Derivation of shape parameters \( m, n \) and \( \eta \) of the retention and hydraulic conductivity curves respectively using Haplic Acrisol, Umbria Fluvisols and Ferric Acrisol soils data

Figure 1: Model and observations curves of the optimizing particle-size distribution function on soils of Allada Plateau (Haplic Acrisol, Umbria Fluvisols and Ferric Acrisol)

Hydrodynamic shape parameters \( m, n \) and \( \eta \) were determined by optimization of the particle size distribution function (Eq.26) by the technique of least squares of Marquardt Levenberg written in
the R program. The graphics shape parameters optimization are those shown in Figure 1. This program takes as input the cumulative particle size curve diameters and associated vectors, dry soil density and the initial vector of values to optimize. As output, the code gives the optimized values of m, n and \( \eta \) from associated parameters \( \eta \), N, M, s, and \( \eta \) M pm. Table 1 shows the hydrodynamic shape parameters optimized for the soils of the study area that are haplic Acrisols (ACho), umbric Fluvisols (FLu) and ferric Acrisols (ACfo). The algorithm relating to particle size analysis and data described by the equations from (26) to (32) of the previous sections has helped to write this optimization code of the parameters in Table 1.

<table>
<thead>
<tr>
<th>Profils_Profondeurs</th>
<th>Type de sol</th>
<th>( \text{dg (\mu m)} )</th>
<th>N</th>
<th>M</th>
<th>MN</th>
<th>s</th>
<th>m</th>
<th>( \eta )</th>
<th>pm</th>
<th>( \eta ) M</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACho_SOCV11_50</td>
<td>ACho</td>
<td>2098.259</td>
<td>2.279</td>
<td>0.122</td>
<td>0.279</td>
<td>2</td>
<td>0.412</td>
<td>3.405</td>
<td>4.422</td>
<td>0.994</td>
</tr>
<tr>
<td>ACho_SOCV19_90</td>
<td>ACho</td>
<td>4164.955</td>
<td>2.189</td>
<td>0.086</td>
<td>0.189</td>
<td>2</td>
<td>0.314</td>
<td>3.014</td>
<td>5.715</td>
<td>0.699</td>
</tr>
<tr>
<td>ACho_SOCV1_90</td>
<td>ACho</td>
<td>4684.902</td>
<td>2.207</td>
<td>0.094</td>
<td>0.207</td>
<td>2</td>
<td>0.336</td>
<td>3.014</td>
<td>4.972</td>
<td>0.758</td>
</tr>
<tr>
<td>ACho_SOCIV1_50</td>
<td>ACho</td>
<td>2045.758</td>
<td>2.198</td>
<td>0.091</td>
<td>0.198</td>
<td>2</td>
<td>0.325</td>
<td>2.966</td>
<td>5.069</td>
<td>0.728</td>
</tr>
<tr>
<td>ACho_SOCV19_50</td>
<td>ACho</td>
<td>890.204</td>
<td>2.517</td>
<td>0.205</td>
<td>0.517</td>
<td>2</td>
<td>0.574</td>
<td>4.703</td>
<td>3.739</td>
<td>1.716</td>
</tr>
<tr>
<td>ACho_SOCV3_90</td>
<td>ACho</td>
<td>2668.638</td>
<td>2.195</td>
<td>0.089</td>
<td>0.195</td>
<td>2</td>
<td>0.322</td>
<td>2.951</td>
<td>5.103</td>
<td>0.719</td>
</tr>
<tr>
<td>ACho_SOCIV1_50</td>
<td>ACho</td>
<td>2091.740</td>
<td>2.279</td>
<td>0.123</td>
<td>0.279</td>
<td>2</td>
<td>0.413</td>
<td>3.408</td>
<td>4.149</td>
<td>0.996</td>
</tr>
<tr>
<td>ACho_SOCIV2_50</td>
<td>ACho</td>
<td>976.261</td>
<td>2.467</td>
<td>0.189</td>
<td>0.467</td>
<td>2</td>
<td>0.549</td>
<td>4.437</td>
<td>3.820</td>
<td>1.573</td>
</tr>
<tr>
<td>FLu_SOCV3_50</td>
<td>FLu</td>
<td>1875.479</td>
<td>2.352</td>
<td>0.149</td>
<td>0.352</td>
<td>2</td>
<td>0.475</td>
<td>3.809</td>
<td>4.105</td>
<td>1.226</td>
</tr>
<tr>
<td>FLu_SOCV15_90</td>
<td>FLu</td>
<td>1026.517</td>
<td>2.596</td>
<td>0.229</td>
<td>0.596</td>
<td>2</td>
<td>0.609</td>
<td>5.120</td>
<td>3.641</td>
<td>1.939</td>
</tr>
<tr>
<td>FLu_SOCVII3_50</td>
<td>FLu</td>
<td>1563.455</td>
<td>2.484</td>
<td>0.195</td>
<td>0.484</td>
<td>2</td>
<td>0.558</td>
<td>4.529</td>
<td>3.790</td>
<td>1.623</td>
</tr>
<tr>
<td>FLu_SOCV1_90</td>
<td>FLu</td>
<td>2319.977</td>
<td>2.270</td>
<td>0.119</td>
<td>0.270</td>
<td>2</td>
<td>0.404</td>
<td>3.358</td>
<td>4.472</td>
<td>0.967</td>
</tr>
<tr>
<td>FLu_SOCIII3_90</td>
<td>FLu</td>
<td>478.335</td>
<td>2.399</td>
<td>0.166</td>
<td>0.399</td>
<td>2</td>
<td>0.508</td>
<td>4.065</td>
<td>3.968</td>
<td>1.369</td>
</tr>
<tr>
<td>ACfo_SOCII3_50</td>
<td>ACfo</td>
<td>12661.6</td>
<td>2.144</td>
<td>0.067</td>
<td>0.144</td>
<td>2</td>
<td>0.253</td>
<td>2.678</td>
<td>5.947</td>
<td>0.541</td>
</tr>
<tr>
<td>ACfo_SOCIII3_90</td>
<td>ACfo</td>
<td>2486.407</td>
<td>2.255</td>
<td>0.112</td>
<td>0.253</td>
<td>2</td>
<td>0.387</td>
<td>3.262</td>
<td>4.583</td>
<td>0.910</td>
</tr>
<tr>
<td>ACfo_SOCII1_50</td>
<td>ACfo</td>
<td>1205.985</td>
<td>2.414</td>
<td>0.171</td>
<td>0.415</td>
<td>2</td>
<td>0.517</td>
<td>4.149</td>
<td>3.931</td>
<td>1.416</td>
</tr>
</tbody>
</table>

Assessment of the accuracy of the optimization method for normalization parameters \( K_s \) and \( \alpha_s \) using Haplique Acrisols, Umbric Fluvisols and Ferrique Acrisols soils data for one-dimensional infiltration

The optimization of scale parameters was made according to steps from 1 to 7 with the equations from (33) to (43) of the algorithm as described in the section devoted there above. It gives the bounds of scale parameters \( K_s \) and \( \alpha_s \) and subsequently \( h_s \) from GA and TP models (Eqn (24) and (25)). The R code used sets the GA and TP models as functions that take as inputs in this order, the infiltration data vector \( i_{10} \) and associated times \( t \), \( \theta_{0} \) experimentally determined (Eqn. (34)), \( \theta_{0} \) experimentally determined (Eqn. (33)), \( n \) (Table 1), \( \eta \) (Table 1) and \( h_{surf} \) (Eqn. (33)). As outputs, the code gives the values of \( K_s \) and \( \alpha_s \) and subsequently \( h_s \), function of sorptivity S and \( \alpha_s \) (Eqts. (7), (8) and (19)). The error RMSE on the adjustment defined by equation (42) is used to assess the quality of optimization. Table 1 shows for Haplic Acrisols, Umbric Fluvisols and Ferric Acrisols, the parameters \( n \) and \( m \) of the characteristic water retention curve of van Genuchten (1980) and the \( \eta \) parameter of the characteristic hydraulic conductivity curve of Brooks and Corey (1964). The parameters N and M of the semi-physical formula of the particle-size distribution curve allow the calculation of \( n \), \( m \) and \( \eta \). They are bound to N and M by pm and \( \eta \) M pm known as soils specific shape indexes by the theory of similarity of forms developed by Arya and Paris (1981) and Haverkamp and Parlange (1986). Tables 2, 3 and 4 below present values of \( K_s \), S, \( \alpha_s \) and \( h_s \) obtained by optimizing on dimensional infiltration data (the shape parameters, the initial water content and the saturated water content being known). Figures 2, 3 and 4 below present dimensional and dimensionless curves of optimization of the scale parameters of by fitting the dimensional infiltration data on Green & Ampt (GA) and Talsma & Parlange (TP) models.
Models of Green & Ampt (1911) and Talsma & Parlange (1972) are simple solutions, less general, approaching the phenomenon of infiltration. The curves given by these two solutions are the two extreme curves which limit any phenomenon of infiltration (Smith and Parlange, 1978; Parlange et al. 1982). In the case of soils in our study area, only Acrisols Haplic of SOCI2_50 sites (ACHo_SOCI2_50) and SOCIV1_50 sites (ACHo_SOCIV1_50) provide optimization results from the infiltration data conforming to the above postulate. The Haplic Acrisols of SOCIV1_50 sites (ACHo_SOCIV1_50), SOCI4_90 sites (ACHo_SOCI4_90) and SOCV2_90 sites (ACHo_SOCV2_90) have their optimized curves outside the limits of GA and TP models (Figures 5 below), therefore optimized values of scale parameters excluding between the limit values of GA and TP (table 2 below). It should be noted that the infiltration curve was contained within the envelope curves in the early hours of infiltration (approximately 1 hour on a total of 2h30 infiltration time). But the rest of the infiltration time, GA and TP models have not reproduced these soils.

Braud et al. (2005) conducted similar work on two soils with physical and hydrodynamic properties very different, the Grenoble Sand and Yolo Light Clay. The results of optimization of scale parameters and subsequently as part of their study were very satisfactory. The optimization curves of Grenoble Sand and Yolo Light Clay infiltration data were well bounded by the envelope curves of GA and TP, giving optimized scale parameter values between the intervals of GA and TP values.

The other soils, namely Umbric Fluvisols (Figure 3 below) and Ferric Acrisols gave at parameters optimization, curves outside the envelope curves of GA and TP. The Ferric Acrisols of SOCI1_50 site (ACfo_SOCI1_50) were exceptions. Their optimization curve is not contained in the envelope curves of GA and TP. But we see that Green & Ampt model reproduced perfectly infiltration in these soils (Figure 4), and consequently the optimized values for these soils are very close to GA limit values (Table4).

The original solution of Green & Ampt considers only the soil structure parameters, while that of Talsma & Parlange rather prefers the texture (De Condappa, 2002). This justifies the fact that infiltration graph on any soil must be contained between the curves of GA and TP, becoming the envelope curves.

The equation of Green & Ampt (1911) is a simplified solution of the Richards equation. The original version was developed in 1911 with an assumption of positive surface head and for soils of very specific configurations. Since then, the equation form has been slightly modified by widespread concern (de Condappa, 2002).

Haverkamp et al. (1994) introduced into the formalism of Green & Ampt equation the influence of texture via the S parameter called sorptivity, integral characteristic of soils function of cp (Philip, 1957). Accordingly, we can apply Eqt. (13) or (14) resulting whatever the soil. However, when we represent the Eqt. (13) curve or (14), it is still on the curve of Green & Ampt although the soil is not of Green & Ampt configuration, it means that we have for a given \( K_s \) a value of infiltration slightly excessive.

The results provided by this solution are quite good as this simple model describes quite accurately what happens in reality on any soil: the phenomenon of infiltration depends mainly on the soil structure.

The widespread equation of infiltration of Haverkamp & Parlange (1990) is also a solution of the Richards equation (1931). The equation of Haverkamp & Parlange contains among other solutions, the Talsma & Parlange solution. It is reduced to the Talsma & Parlange (1972) model for the case where \( \beta = 1 \) and \( \gamma = 0 \) (see Eqts. (13) and (14)).
**Figure 2:** (To the left) Comparison of fitted observed cumulative infiltration as a function of time of ACho_SOCV1_50. (To the right) Non-dimensional infiltration as a function of non-dimensional time fitted using the Green & Ampt model, the Talsma & Parlange model and observed cumulative infiltration.

**Table 2:** Optimized values of normalization parameters and scaling factor of ACho_SOCV1_50

<table>
<thead>
<tr>
<th>ACho_SOCV1_50</th>
<th>Référence (Valeurs de laboratoire)</th>
<th>One-dimensional GA</th>
<th>One-dimensional TP</th>
<th>Relative error GA / %</th>
<th>Relative error TP / %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_0 / \text{mm s}^{-1}$</td>
<td>1.07E-04</td>
<td>0.01569</td>
<td>0.0192</td>
<td>-99.301</td>
<td>-99.443</td>
</tr>
<tr>
<td>$K_1 / \text{mm s}^{-1}$</td>
<td>-</td>
<td>0.0153</td>
<td>1.405</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$S(\theta_0, \theta_s) / \text{mm}^2 \text{s}^{-1}$</td>
<td>30.40</td>
<td>1.161</td>
<td>43.345</td>
<td>40.219</td>
<td>-24.414</td>
</tr>
<tr>
<td>$c_1 / \text{mm}^{-1}$</td>
<td>-89.62</td>
<td>-133.900</td>
<td>-123.481</td>
<td>33.066</td>
<td>27.418</td>
</tr>
<tr>
<td>$l_{vo} / \text{mm} / (l_{vo})$</td>
<td>-</td>
<td>1.0529</td>
<td>1.0529</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\text{RMSE} / \text{s}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Figure 3:** (To the left) Comparison of fitted observed cumulative infiltration as a function of time of FLu_SOCV3_90. (To the right) Non-dimensional infiltration as a function of non-dimensional time fitted using the Green & Ampt model, the Talsma & Parlange model and observed cumulative infiltration.
Table 3: Optimized values of normalization parameters and scaling factor of FLu_SO7CV3_90

<table>
<thead>
<tr>
<th>FLu_SO7CV3_90</th>
<th>Référence (Valeurs/laboratoire)</th>
<th>One-dimensional GA</th>
<th>One-dimensional TP</th>
<th>Relative error GA / %</th>
<th>Relative error TP / %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_0 / \text{mm s}^{-1}$</td>
<td>-</td>
<td>0.0765</td>
<td>0.0765</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$K_1 / \text{mm s}^{-1}$</td>
<td>-</td>
<td>0.1056</td>
<td>0.1124</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$S^2(\theta_0, \theta_s) / \text{mm}^2 s^{-1}$</td>
<td>-</td>
<td>6.908</td>
<td>12.0472</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$a / \text{mm}^{-1}$</td>
<td>10.134</td>
<td>37.600</td>
<td>59.803</td>
<td>-73.048</td>
<td>-83.054</td>
</tr>
<tr>
<td>$h_{veg} / \text{mm (B_{ega}}$</td>
<td>-15,0011289</td>
<td>105.915</td>
<td>-174.699</td>
<td>-85.836</td>
<td>-91.413</td>
</tr>
<tr>
<td>RMSE/s</td>
<td>-</td>
<td>13.3203</td>
<td>13.3203</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 4: (To the left) Comparison of fitted observed cumulative infiltration as a function of time of ACfo_SO7CII_50. (To the right) Non-dimensional infiltration as a function of non-dimensional time fitted using the Green & Ampt model, the Talsma & Parlange model and observed cumulative infiltration.

Table 4: Optimized values of normalization parameters and scaling factor of ACfo_SO7CII_50

<table>
<thead>
<tr>
<th>ACfo_SO7CII_50</th>
<th>Référence (Valeurs/laboratoire)</th>
<th>One-dimensional GA</th>
<th>One-dimensional TP</th>
<th>Relative error GA / %</th>
<th>Relative error TP / %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_0 / \text{mm s}^{-1}$</td>
<td>5.06E-05</td>
<td>0.0442</td>
<td>0.0442</td>
<td>-99.897</td>
<td>-99.908</td>
</tr>
<tr>
<td>$K_1 / \text{mm s}^{-1}$</td>
<td>-</td>
<td>0.0494</td>
<td>0.0553</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$S^2(\theta_0, \theta_s) / \text{mm}^2 s^{-1}$</td>
<td>-</td>
<td>2.0018</td>
<td>3.5347</td>
<td>-27.112</td>
<td>-54.939</td>
</tr>
<tr>
<td>$a / \text{mm}^{-1}$</td>
<td>15.574</td>
<td>21.367</td>
<td>34.562</td>
<td>-36.441</td>
<td>-65.281</td>
</tr>
<tr>
<td>$h_{veg} / \text{mm (B_{ega}}$</td>
<td>-39,243</td>
<td>-61.743</td>
<td>-113.029</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>RMSE/s</td>
<td>-</td>
<td>2.7251</td>
<td>2.7251</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

As we have seen, the solution of Talsma & Parlange with that of Green & Ampt, confine any phenomenonofinfiltration.

For a given $K_0$, the solution of Talsma & Parlange provides default values for infiltration. This solution takes into account the soil texture, and considers non-saturated part of the flow, while that of Green & Ampt before generalization considered a saturated part (volumetric water content profile in "marchofstairs").
It is important to emphasize that the analyzes in this study were made essentially from the dimensionless equations, results from dimensional equations being analyzed when necessary. The scaling is to make the above equations independent of spatial scale: the so-called "scaling approach". The scaling approach defines scaling parameters that contain the specific soil characteristics studied at spatial scale that means: texture, structure, boundary and initial conditions.

The scaling parameters \( \alpha \) allow to remove to dimensional infiltration data \( \Delta I_3 \) and \( \Delta i_3 \) the specific characteristics of soils considered, thus giving the values of dimensionless infiltration data \( \Delta I_3^* \) and \( \Delta i_3^* \) invariant according to the scale of the study.

Expressions (24) and (25) define each a unique class of dynamic similarity to any phenomenon of infiltration (de Condappa, 2002). In other words, if one performs infiltration tests on different soils, all the experimental points will be found whatever the scale on the curve (24) or that of (25) depending on the model (Haverkamp et al., 1998a).

Two applications were developed by de Condappa (2002) to illustrate this scaling theory. The first was performed on cumulative one-dimensional flow infiltration data for soils with contrasting characteristics:

- a sandy soil sample of Grenoble, tests were carried out on a laboratory column (Touma et al., 1984).
- and a clay soil, the Yolo Light Clay whose data are from a numerical simulation (Barry et al., 1995).

From the available physical parameters of these soils, the experimental values were fitted on Talsma & Parlange equation (25) and the scales parameters \( K_s, \alpha\_s \) and subsequently \( h_g \) were calculated.

In both considered soils, the fit is very satisfactory, and when calculating the dimensionless values, knowing \( \alpha_s \) and \( \alpha_i \), they are on the curve of Eq. (25). Both infiltration tests, carried out on two very different soils (sand and clay soil) are thus on the same dimensionless curve.

Comparing the work of Braud et al. (2005) to that of de Condappa (2002) on aspects relating to the question of scaling reveals that the approach to make dimensionless equations is the same but the results targeted by the authors are not the same. In the first case, it is to show that all phenomena of infiltration is bounded by the GA and TP models.

In the second case, it has been shown that when scaling limits equations of GA and TP, the experimental points of infiltration test on a soil anywhere it is taken must be all on either of curves representing boundaries models.

The results in both cases of scaling are not contradictory but complementary.

This study applied to its soils the scaling theory as Braud et al (2005) have discussed. But earlier, it was found that for all the soils consist of Haplic Acrisols, Umbric Fluvisols and Ferric Acrisols, GA and TP models did not reproduce well the hydrodynamic behavior. However the fitting error on all soils is less than 15%. The reasons behind the less satisfactory quality of the modeling can be related to the nature of the soil, to basic hydrodynamic models, the optimization program, etc.

The analysis results made in the laboratory were used for reference in some cases of soils to analyze the scale parameters optimized from the GA and TP models.

Soria (2002 and 2003), de Condappa (2002) and Braud (2005) had used the values of physical and hydrodynamic parameters of GRIZZLY database as a reference in the analysis of a similar work. Soils with parameter values were derived from the base were sandy soils, the Grenoble Sand and clay soils, the Yolo Light Clay mentioned above.

In the case of soils in this study, the error RMSE on the fittings is less than 15%. This means that the optimization technique by inversion adopted in this study is better than traditional techniques. Braud et al (2005) have come to the same conclusion with an error RMSE on soil optimization less than 30%.

Compared to reference values, GA and TP models have overestimated the scale parameters \( K_s \), \( h_g \) and even the scaling factor \( \alpha_i \). The relative errors are around 99% for the \( K_s \) determined from GA and TP (Tables 2, 3 and 4) and vary between 13.48% and 91% for \( h_g \) determined from TP (between 33.06% and 89% for \( h_g \) determined from GA). The optimal values of \( K_s \) and \( h_g \) were determined on soils in situ, so in the natural boundary and initial conditions while the reference values were
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determined on laboratory samples where the boundary conditions are often imposed and initial conditions little influenced. Given these considerations, the optimized values from GA and TP seem to be more realistic.

Assessment of the accuracy of the optimization method for \( K_s \) and \( \delta I \) on simulated values (three-dimensional infiltration)

The three-dimensional analysis is assessed by using infiltration data obtained by Beerkan method.

Haverkamp et al (1994) adjusted the value of the one-dimensional infiltration \( II \) of the previous section by adding the term \( \delta \) (Eq.17) due to the three-dimensional effect. The axisymmetric phenomenon (lateral flow) observed at the infiltration test in situ has led to consider the infiltration resulting data as being three-dimensional infiltration data. Infiltration data used in one-dimensional analysis above were generated by applying \( \delta \) to the three-dimensional data (Eq.16). The same soil namely Acrisols Haplic, Umbric Fluvisols and Ferric Acrisols are soils from which the data were obtained.

A duration of 8 units of dimensionless time allowed to have 18 experimental points, which corresponds to an effective duration of infiltration of about 117 minutes for Haplic Acrisols of SOCV1_50 site (Figure 2), and more 150 minutes for Haplic Acrisols of SOCI2_50 site.

For Umbric Fluvisols of SOCV3_90 site (Figure 3), 7 units of dimensionless time allowed to have 84 experimental points for an effective duration of approximately 115 minutes and 123 minutes for Umbric Fluvisols of SOCI5_90 site.

Concerning Ferric Acrisols of SOCII1_50 site (Figure 4), 7.5 units of dimensionless time yielded 25 experimental points with a total period of effective infiltration of approximately 117 minutes and 150 minutes for Ferric Acrisols of SOCI3_50 site.

The cumulative values of the three-dimensional infiltration are 153 mm for the Haplic Acrisols of SOCV1_50 site, 705.5 mm for Umbric Fluvisols of SOCV3_90 site and 425 mm for the Ferric Acrisols of SOCII1_50 site.

Braud et al (2005), for the Grenoble sandy soil, found 50 experimental points for a dimensionless time span of 2.5 units, corresponding to an effective duration (dimensional time) of 23 minutes and more than 200 days for the soil clay of Grenoble, the Yolo Light Clay.

The optimization was applied to three-dimensional data to determine \( K_s \) and \( h_g \) from the scaling factor \( \delta \). To this end, the output parameters of the 1D optimization and \( K_s \) values were used as input to the 3D optimization. As 1D optimization in the context of this study was not satisfactory, this has influenced the optimized parameter values in 3D, as well as graphics that result so the optimization curves were not very different from those obtained in 1D.

That is why three-dimensional analysis of this section refers to the same graphics above.

As against the three-dimensional analysis of Braud et al (2005) on the sandy soil and clay soil of Grenoble gave relatively successful results. Good agreement was obtained, with estimated errors less than 20%, with the exception of the three-dimensional analysis with the GA model. The analysis was more accurate when the GA model was used, including the fact that the GA model failed to estimate properly the parameter \( \delta \) that integrates the three-dimensional effect. Which may be one of the causes of the unsatisfactory quality of the 1D optimization study. TP model, in contrast, has estimated the parameter.

The Figure 5 below show the corresponding optimization curves. Infiltration data called references in the context of the work of Braud et al (2005), is contained between the curves representing GA and TP models. Confirming that both models can generally be used as limit curves.
Figure 5: (a) Upper curves: comparison of fitted and reference cumulative three-dimensional infiltration as a function of time. The lower curves present the corresponding reference one-dimensional infiltration and the one-dimensional infiltration derived with the parameters estimated from the Green & Ampt and Talsma & Parlange models using the three-dimensional analysis. (b) Dimensionless infiltration as a function of dimensionless time fitted with the Green & Ampt model, the Talsma & Parlange model and the reference values. Results are given for the Yolo Light Clay (simulated three-dimensional infiltration values, Braud et al., 2004)

Esteves (2000) conducted analyses of data from infiltration test obtained in situ on three types of soil in Niger, taking into account the three-dimensional nature of the flow.

Each of these soils was crusted with hydraulic properties differ from the rest of the soil area. These crusts, thick of a few millimeters, formed after rainfall or runoff and are low permeability. The tests were conducted during the dry season, with initially dry soils, therefore $\theta_0$ et $K_0$ were assumed to be zero.

Unlike the one-dimensional infiltration, the flow was considered axisymmetrical around the circular infiltrometer and experimental values were adjusted to the equation of Green & Ampt (GA) where the parameter $\delta$ has been integrated. The adjustment was carried out for six (06) infiltration tests (two infiltration tests for each soil). The curves of adjustment represented on the same graph are as follows (Figures 6 et 7).
Figure 6: Adjusted Equations (Adj curves) and experimental points (Exp points) of infiltration tests on soil from Niger (de Condappa and Soria, 2002)

![Graph 1: Adjusted Equations and Experimental Points](image1)

Figure 7: Dimensionless experimental values for some soils of Niger, and fitted curve of Eqt. (26) (De Condappa and Soria, 2002)

![Graph 2: Dimensionless Infiltration Values](image2)

Calculating the dimensional values for each infiltration tests yielded the graph below (de Condappa, 2002):
The adjustment in the context of the work of Esteves (2000) has been satisfactory since the distribution of experimental points follows the curve Eq. (24), except for some short time values to the Erod04b soil.

Furthermore, the author suggested to correct "3D effect" and avoid getting excessive values for $K_p$, knowledge of the volumetric saturated water content at saturation $\theta_s$ is necessary. In this case, this data was available; de Condappa (2002) developed procedure in the opposite case.

**Conclusion**

Soils in our study area subject to this analysis are of three types namely Acrisols Haplic, Umbric Fluvisols and ferric Acrisols.

The analysis was done on two types of data, particle size data analysis and infiltration data, the first experimentally obtained in the laboratory and the second in situ from the method of Beerkan.

The optimization results on particle size data analysis were used, with certain physical parameters determined in the laboratory and used as inputs to the optimization program that allowed the dimensional analysis of infiltration data. $K_s$ resulting value is used as input for the optimization of data in three-dimensional analysis.

The description of the hydrodynamic behavior of Haplic Acrisols, the Umbric Fluvisols and ferric Acrisols made from models of Green & Ampt (1911) and Talsma & Parlange (1972) proved exceptionally satisfactory for some cases of soils but unsatisfactory for all the soils of the study area. Errors on fittings around 15% for the whole soil of this study, which is well below the 30% error that Braud et al. (2005) found for the whole of their soils in the frame of similar study where they conclude good their fitting results. For most soils of this study, the three optimization curves were not reproduced so that the observations curve is contained in the GA and TP envelope curves or overlaps one on another as postulate the laws of flow in the unsaturated zone. This law has been respected for most soils of Braud et al (2005), this has been reflected in the above conclusion of these authors as well as errors on the adjustments in this study were far less than their own, the adjustment was found good. But it is clear that the optimization results could influence the fact that most of these authors have worked on synthesized soils (sandy soil and clay soil of Grenoble). Indeed, infiltration data from these synthesized soils and considered as reference data were generated from an equation developed under ideal conditions, the explicit equation of Barry et al (1995) which only need some physical and hydraulic parameters of soils considered and determined under ideal conditions of laboratory. Concerning soils of this study, infiltration data have not been generated but are obtained in situ through infiltration tests implemented by the method of Beerkan.

Nevertheless, the above assumption has been respected for soil of rim plateau of high slopes namely ferric Acrisols of SOCI1_50 site (ACfo_SOCI1_50) and Gleyic Acrisols of SOCI2_50 site (ACgo_SOCI2_50), located respectively in middle of slope and to the bottom of the slope (Figures 9 and 10).

The ACfo_SOCI1_50 has a particle-size profile composed of 18.4% of silt, 39.7% of clay and 40.38% of sand. The textural triangle of Durand (1960) class this soil clay loam and gives it a fine textured soil, so ferric Acrisols of SOCI1_50 site are fine-textured clay. In addition, the proportions of sand and clay in these soils are between 20% and 80%, and that of silt is less than 20%. According to Soclo et al (2015), the ACfo_SOCI1_50 are bimodal soils. This characteristic greatly influences the hydrodynamic behavior of soils.

Regarding the Gleyic Acrisols of SOCI2_50 site, their particle-size profile presents 17.75% of silt, 16.80% of clay and 65.39% of sand. It is a sandy loam from the classification of Durand (1960) which gives a coarse textured soil. The projection of percentages of these granular fractions in the textural triangle USDA (1960), with associated bimodal areas gives gleyic Acrisols, unimodal soils (Soclo et al., 2015). This character affects the hydrodynamic nature of soils but it facilitates such studies and remains an indispensable and necessary condition to check before any modeling on soils. Most hydrodynamic models, especially those of Van Genuchten (1980) and Brooks and Corey (1960) at the base of laws and theory of flow in unsaturated zone rely on the condition of soils unimodality.
Figure 8: (to the left) and Figure 9 (to the right): (To the left of each figure) Comparison of fitted observed cumulative infiltration as a function of time of ACfo_SOCII1_50 and ACgo_SOCII2_50. (To the right of each figure) Non-dimensional infiltration as a function of non-dimensional time fitted using the Green & Ampt model, the Talsma & Parlange model and observed cumulative infiltration.

Indeed, in the case of ferric Acrisols of SOCII1-50 site of figure 8, the curve of infiltration data is not bounded by the limits curves of Green & Ampt (GA) and Talsma & Parlange (TP). But it overlaps well with the Green & Ampt model. Unlike the Talsma & Parlange model that seemed at first to fit the data but who departed early. The GA model therefore correctly describes the hydrodynamic behavior of ferric Acrisols of SOCII1-50 site and gives an error on data adjustment and optimization of Ks values, hh and $\alpha_f$ of 2.72%.

For gleyic Acrisols of SOCII2_50 site of Figure 9, the observations curve is not limited by the envelope curves but it is described by the GA model for a relatively long time before starting to diverge towards the end of the process. This convergence of observations with the GA model has certainly come to the times when the flow is transient and divergence was observed when flow began the steady state. The GA model has reproduced the gleyic Acrisols of SOCII2_50 site. The Talsma & Parlange model, contrariwise, did not reproduce in general the soils of the study area (Figures 8 and 9).

In both cases, the models do not specify the flow regimes as in the optimization method of Labassaltère et al (2006) that we will use for its practical and very effective appearance.

Braud et al (2005) concluded that the TP model generally gives more accurate results than the GA model, especially for clay.

But in this study, it was found the opposite with the TP model as described above. The model GA, contrariwise, correctly described the ACfo_SOCII1_50 and ACgo_SOCII2_50 which are, respectively, clay soils and sandy soils. Confirming that the introduction of the texture effect in formalizing of GA model via sorptivity makes this model, a model that describes any soils.

de Condappa (2002) seems to say the same as the aforementioned authors mentioning, we quote "The original solution of Green & Ampt only consider a soil structure parameters, while that of Talsma & Parlange rather favors texture". But the generalization to any soil, the GA model now considered in addition to the structure parameters, the texture parameters. It is rather fine-textured settings of soils that have been considered by the TP model as part of the work of Braud et al (2005). Soria (2003) showed the influence of the following parameters on the optimization of infiltration curves: number of points, period, point spacing and volume content of initial water.
Braud et al (2005) have made a sensitivity analysis of the robustness of the method for one-
dimensional infiltration to quantify the errors induced by the number of points, period, spacing points
and the volume initial water content. A minimum of 10 measurement points for a given infiltration
curve is required according to these authors for accurate estimates of $K_s$ and $h_g$, with a dimensionless
time of about 2 units for sand and 0.5 for clay.

Soria (2003) studies also recommends 10 measurement points for a given infiltration curve for
accurate estimates of $K_s$ and $h_g$, with a dimensionless time of 2 units for accurately estimating of $K_s$
in both cases of soil (clay and sand) and 2.5 units for an accurate estimate of $h_g$ in both cases of soil.

The condition of having at least 10 measurement points for infiltration curves and time
dimensionless value at least equal to those found by the above mentioned authors, is respected in this
study to minimize errors induced by the number of points measurement and duration on optimized
values of $K_s$ and $h_g$.

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